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INVENTORY MODEL FOR IMPERFECT QUALITY ITEMS UNDER DIFFERENT DETERIORATION RATES AND SHORTAGES CONSIDERING PRICE AND TIME DEPENDENT DEMAND WITH INLFATION AND PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT

Some of the assumptions of the classical inventory models are not fulfilled with current situations. One of the assumptions is that the items received in a lot are all perfect items. Many times it happens that items received in a lot are not of 100% good quality. Some of the items are of defective quality in the lot received. Another assumption is that as soon as items are received, payments are made. In today's reality situation, the supplier allows certain fixed period known as permissible delay for payment to the retailer for settling the amount of items received. Keeping this reality, a deterministic inventory model with imperfect quality items is developed when deterioration rate is different during a cycle. Here it is assumed that demand is a function of time and price. Shortages are allowed and completely backlogged. Numerical example is taken to support the model. Sensitivity analysis is also carried out for parameters.

Key Words: Inventory model, Varying Deterioration, Time dependent demand, Price dependent demand, Defective items, Shortages, Inflation, Permissible delay

1. INTRODUCTION

Many researchers have investigated on inventory models for deteriorating items in recent years. In real life, many items like vegetables, fruits, medicines, volatile liquids, etc. either deteriorate or become obsolete in the course of time. Therefore, if the rate of deterioration is not sufficiently low, its impact on modeling of such an inventory system cannot be ignored. An inventory model with constant rate of deterioration was considered by Ghare and Schrader (1963). The model was extended by Covert and Philip (1973) using variable rate of deterioration. By considering shortages, the model was further extended by Shah and Jaiswal (1977). Mandal and Phaujdar (1989) presented an inventory model for stock dependent consumption rate. Haiping and Wang (1990) studied an economic policy model for deteriorating items with time proportional demand. Patel and Parekh (2014) developed an inventory model with stock dependent demand under shortages and variable selling price. Sheikh and Patel (2015) developed an inventory model with stock and price dependent demand under shortages. Other research work related to deteriorating items can be found in, for instance [Raafat (1991), Goyal and Giri (2001), Ruxian et al. (2010)].

Many times it happens that some of the items received are not of good quality i.e. some of the items received are defective items. An inventory model for defective items was developed by Lee and Rosenblatt (1985). Cheng (1991) developed a model of imperfect production quantity by establishing relationship between demand dependent unit production cost and imperfect production process. Salameh and Jaber (2000) developed an inventory model in which items received are of defective quality and after 100% screening, imperfect items are withdrawn from the inventory and sold at a discounted price. Goyal and Barron (2002) considered a simple approach for determining economic production quantity model for imperfect quality items. Wee et al. (2007) extended the model developed by Salameh and Jaber (2000) by allowing shortages. Yassine et al. (2014) considered an EPQ model with disaggregation and consolidation of imperfect quality shipments. Patel and Patel (2012) developed an EOQ model for deteriorating items with imperfect quantity items. An inventory model for optimum order quantity of product batches that contains defective items with percentage nonconforming following a known probability density function was developed by Vishkaei et al. (2014).

In classical EOQ model it is assumed that retailer must pay off as and when items are received. But in today's competitive market that will not be always true. The supplier often offers his retailer certain delay in time period for making payment for the items he has received. It is an effective way of attracting new customers. An economic order quantity model under the condition of permissible delay in payments was developed by Goyal (1985). Goyal's (1985) model was extended by Aggarwal and Jaggi (1995) to consider the deteriorating items. The related works are found in [Chung and Dye (2002), Salameh et al. (2003), Chung et al. (2005), Chang et al. (2008)].

The effect of inflation play important role in practical situations. Buzacott (1975) and Mishra (1975) simultaneously developed inventory model with constant demand and single inflation rate for all associated costs. Mishra (1979) considered different inflation rate for different costs associated with inventory model with constant rate of demand. Liao et al. (2000) developed an inventory model for stock dependent consumption and permissible delay in payment under inflationary conditions. Singh (2011) considered an EOQ model with linear demand and permissible delay in payments. The effect of inflation and time value of money were also taken into account. An inventory model with inflation and permissible delay in payments was considered by Patel and Patel (2013). Muniappan et al. (2015) developed an EOQ model for deteriorating items with inflation and delay in payments considering time dependent deterioration rate.

Generally the products are such that initially there is no deterioration. Deterioration starts after certain time and again after certain time the rate of deterioration increases with time. Here we have used such a concept and developed the deteriorating items inventory model.

In this paper we have developed an inventory model for imperfect quality items with different deterioration rates under price and time dependent demand. Shortages are allowed and completely backlogged. Numerical example is provided to illustrate the model. Sensitivity analysis for major parameters is also carried out.

2. ASSUMPTIONS AND NOTATIONS

NOTATIONS:

The following notations are used for the development of the model:

D(t) : Demand rate is a function of time and price ($a+bt-\rho p$, $a>0$, $0<b<1$, $\rho>0$)
c : Purchasing cost per unit
p : Selling price per unit
d : defective items (%)
1-d : good items (%)
λ : Screening rate
SR : Sales revenue
A : Replenishment cost per order for
z : Screening cost per unit
p_d : Price of defective items per unit
h(t) : Variable Holding cost ($x + yt$, $x>0$, $0<y<1$)
M : Permissible period of delay in settling the accounts with the supplier
I_e : Interest earned per year
I_p : Interest paid per year
R : Rate of inflation
c_2 : Shortage cost per unit
t_1 : Screening time
T : Length of inventory cycle
I(t) : Inventory level at any instant of time t, $0 \le t \le T$
Q_1 : Order quantity initially
Q_2 : Quantity of shortages
Q : Order quantity
θ : Deterioration rate during $\mu_1 \le t \le \mu_2$, $0< \theta<1$
θt : Deterioration rate during , $\mu_2 \le t \le T$, $0< \theta<1$
π : Total relevant profit per unit time.

ASSUMPTIONS

The following assumptions are considered for the development of the model.

- The demand of the product is declining as a function of time and price.
- Replenishment rate is infinite and instantaneous.
- Lead time is zero.
- Shortages are allowed and completely backlogged.
- The screening process and demand proceeds simultaneously but screening rate (λ) is greater than the demand rate i.e. $\lambda > (a+bt-\rho p)$.
- The defective items are independent of deterioration.
- Deteriorated units can neither be repaired nor replaced during the cycle time.
- A single product is considered.
- Holding cost is time dependent.
- The screening rate (λ) is sufficiently large. In general, this assumption should be acceptable since the automatic screening machine usually takes only little time to inspect all items produced or purchased.
- During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the credit period, the account is settled as well as the buyer pays off all units sold and starts paying for the interest charges on the items in stocks.

3. THE MATHEMATICAL MODEL AND ANALYSIS

At time t=0, a lot size of Q units enters the system. Each lot having a d % defective items. The nature of the inventory level is shown in the given figure, where screening process is done for all the received items at the rate of λ units per unit time which is greater than demand rate. After screening, a portion is used to meet the backlogging items towards previous shortages and initial inventory for period is Q_1. During the screening process the demand occurs parallel to the screening process and is fulfilled from goods which are found to be of perfect quality by the screening process. The defective items are sold immediately after the screening process at time t_1 as a single batch at a discounted price. After the screening process at time t_1 the inventory level will be I(t) and at time t_0, inventory level will become zero due to demand and partially due to deterioration. Shortages occur during the period t_0 to T and of size Q_2 units.

Also here $t_1 = \frac{Q}{\lambda}$ (1)

and defective percentage (d) is restricted to $d \le 1 - \frac{(a+bt-\rho p)}{\lambda}$ (2)

Let I(t) be the inventory at time t ($0 \le t \le T$) as shown in figure.

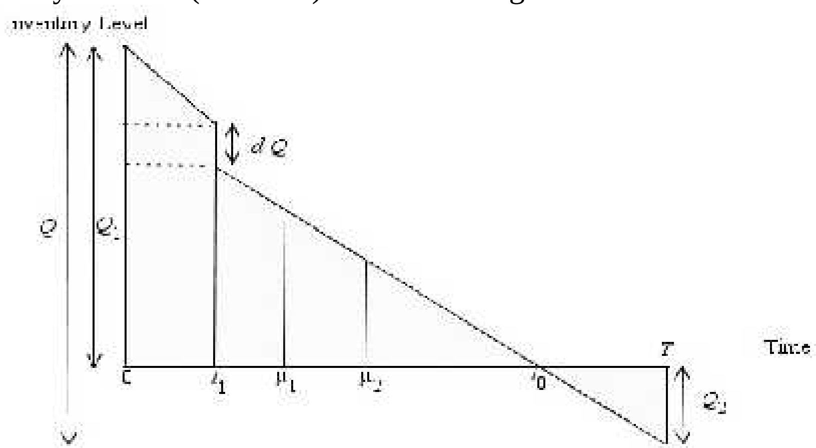


Figure 1

The differential equations which describes the instantaneous states of I(t) over the period (0, T) is given by

$$\frac{dI(t)}{dt} = -(a + bt - \rho p), \qquad 0 \le t \le \mu_1 \qquad (3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt - \rho p), \qquad \mu_1 \le t \le \mu_2 \qquad (4)$$

$$\frac{dI(t)}{dt} + \theta t I(t) = -(a + bt - \rho p), \qquad \mu_2 \le t \le t_0 \qquad (5)$$

$$\frac{dI(t)}{dt} = -(a + bt - \rho p), \qquad t_0 \le t \le T \qquad (6)$$

with initial conditions $I(0) = Q_1$, $I(\mu_1) = S_1$, $I(t_0) = 0$, and $I(T) = -Q_2$.
Solutions of these equations are given by

$$I(t) = Q_1 - (at - \rho pt + \frac{1}{2}bt^2), \qquad (7)$$

$$I(t) = \begin{bmatrix} a(\mu_1 - t) - \rho p(\mu_1 - t) + \frac{1}{2}a\theta(\mu_1^2 - t^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - t^2) + \frac{1}{2}b(\mu_1^2 - t^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - t^3) - a\theta t(\mu_1 - t) + \rho p\theta t(\mu_1 - t) - \frac{1}{2}b\theta t(\mu_1^2 - t^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - t)\right] \tag{8}$$

$$I(t) = \begin{bmatrix} a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2) + \frac{1}{6}a\theta(t_0^3 - t^3) - \frac{1}{6}\rho p\theta(t_0^3 - t^3) \\ + \frac{1}{8}b\theta(t_0^4 - t^4) - \frac{1}{2}a\theta t^2(t_0 - t) + \frac{1}{2}\rho p\theta t^2(t_0 - t) - \frac{1}{4}b\theta t^2(t_0^2 - t^2) \end{bmatrix}. \tag{9}$$

$$I(t) = \left[a(t_0 - t) - \rho p(t_0 - t) + \frac{1}{2}b(t_0^2 - t^2)\right]. \tag{10}$$

(by neglecting higher powers of θ)

After screening process, the number of defective items at time t_1 is dQ.
So effective inventory level during $t_1 \le t \le T$ is given by

$$I(t) = Q_1 - dQ - (at - \rho pt + \frac{1}{2}bt^2). \tag{11}$$

From equation (7), putting $t = \mu_1$, we have

$$Q_1 = S_1 + \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right). \tag{12}$$

From equations (8) and (9), putting $t = \mu_2$, we have

$$I(\mu_2) = \begin{bmatrix} a(\mu_1 - \mu_2) - \rho p(\mu_1 - \mu_2) + \frac{1}{2}a\theta(\mu_1^2 - \mu_2^2) - \frac{1}{2}\rho p\theta(\mu_1^2 - \mu_2^2) + \frac{1}{2}b(\mu_1^2 - \mu_2^2) \\ + \frac{1}{3}b\theta(\mu_1^3 - \mu_2^3) - a\theta\mu_2(\mu_1 - \mu_2) + \rho p\theta\mu_2(\mu_1 - \mu_2) - \frac{1}{2}b\theta\mu_2(\mu_1^2 - \mu_2^2) \end{bmatrix} + S_1\left[1 + \theta(\mu_1 - \mu_2)\right] \tag{13}$$

$$I(\mu_2) = \begin{bmatrix} a(t_0 - \mu_2) - \rho p(t_0 - \mu_2) + \frac{1}{2}b(t_0^2 - \mu_2^2) + \frac{1}{6}a\theta(t_0^3 - \mu_2^3) - \frac{1}{6}\rho p\theta(t_0^3 - \mu_2^3) \\ + \frac{1}{8}b\theta(t_0^4 - \mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0 - \mu_2) + \frac{1}{2}\rho p\theta\mu_2^2(t_0 - \mu_2) - \frac{1}{4}b\theta\mu_2^2(t_0^2 - \mu_2^2) \end{bmatrix}. \tag{14}$$

So from equations (13) and (14), we get

$$S_1 = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right]. \tag{15}$$

Putting value of S_1 from equation (15) into equation (8), we have

$$I(t) = \frac{\left[1+\theta\left(\mu_1-t\right)\right]}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right]$$
$$+\left[\begin{array}{l} a\left(\mu_1-t\right)-\rho p\left(\mu_1-t\right)+\frac{1}{2}a\theta\left(\mu_1^2-t^2\right)-\frac{1}{2}\rho p\theta\left(\mu_1^2-t^2\right)+\frac{1}{2}b\left(\mu_1^2-t^2\right) \\ +\frac{1}{3}b\theta\left(\mu_1^3-t^3\right)-a\theta t\left(\mu_1-t\right)+\rho p\theta t\left(\mu_1-t\right)-\frac{1}{2}b\theta t\left(\mu_1^2-t^2\right) \end{array}\right]. \tag{16}$$

Similarly putting value of S_1 from equation (15) in equation (12), we have

$$Q_1 = \frac{1}{\left[1+\theta\left(\mu_1-\mu_2\right)\right]} \left[\begin{array}{l} a\left(t_0-\mu_2\right)-\rho p\left(t_0-\mu_2\right)+\frac{1}{2}b\left(t_0^2-\mu_2^2\right)+\frac{1}{6}a\theta\left(t_0^3-\mu_2^3\right)-\frac{1}{6}\rho p\theta\left(t_0^3-\mu_2^3\right) \\ +\frac{1}{8}b\theta\left(t_0^4-\mu_2^4\right)-\frac{1}{2}a\theta\mu_2^2\left(t_0-\mu_2\right)+\frac{1}{2}\rho p\theta\mu_2^2\left(t_0-\mu_2\right)-\frac{1}{4}b\theta\mu_2^2\left(t_0^2-\mu_2^2\right) \\ -a\left(\mu_1-\mu_2\right)+\rho p\left(\mu_1-\mu_2\right)-\frac{1}{2}a\theta\left(\mu_1^2-\mu_2^2\right)+\frac{1}{2}\rho p\theta\left(\mu_1^2-\mu_2^2\right)-\frac{1}{2}b\left(\mu_1^2-\mu_2^2\right) \\ -\frac{1}{3}b\theta\left(\mu_1^3-\mu_2^3\right)+a\theta\mu_2\left(\mu_1-\mu_2\right)-\rho p\theta\mu_2\left(\mu_1-\mu_2\right)+\frac{1}{2}b\theta\mu_2\left(\mu_1^2-\mu_2^2\right) \end{array}\right] + \left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right). \tag{17}$$

Using (17) in (7), we have

$$I(t) = \frac{1}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) - \rho p\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{6}\rho p\theta\left(t_0^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) + \frac{1}{2}\rho p\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) \\ - a\left(\mu_1 - \mu_2\right) + \rho p\left(\mu_1 - \mu_2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) + \frac{1}{2}\rho p\theta\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) + a\theta\mu_2\left(\mu_1 - \mu_2\right) - \rho p\theta\mu_2\left(\mu_1 - \mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{array}\right] + \left(a\left(\mu_1 - t\right) - \rho p\left(\mu_1 - t\right) + \frac{1}{2}b\left(\mu_1^2 - t^2\right)\right) \tag{18}$$

Similarly putting t = T in equation (10), we have

$$Q_2 = \left[a\left(T - t_0\right) - \rho p\left(T - t_0\right) + \frac{1}{2}b\left(T^2 - t_0^2\right)\right]. \tag{19}$$

Putting value of Q_1 and Q_2 from equations (17) and (19), we get value of Q.

$$Q = \frac{1}{\left[1+\theta\left(\mu_1 - \mu_2\right)\right]}\left[\begin{array}{l} a\left(t_0 - \mu_2\right) - \rho p\left(t_0 - \mu_2\right) + \frac{1}{2}b\left(t_0^2 - \mu_2^2\right) + \frac{1}{6}a\theta\left(t_0^3 - \mu_2^3\right) - \frac{1}{6}\rho p\theta\left(t_0^3 - \mu_2^3\right) \\ + \frac{1}{8}b\theta\left(t_0^4 - \mu_2^4\right) - \frac{1}{2}a\theta\mu_2^2\left(t_0 - \mu_2\right) + \frac{1}{2}\rho p\theta\mu_2^2\left(t_0 - \mu_2\right) - \frac{1}{4}b\theta\mu_2^2\left(t_0^2 - \mu_2^2\right) \\ - a\left(\mu_1 - \mu_2\right) + \rho p\left(\mu_1 - \mu_2\right) - \frac{1}{2}a\theta\left(\mu_1^2 - \mu_2^2\right) + \frac{1}{2}\rho p\theta\left(\mu_1^2 - \mu_2^2\right) - \frac{1}{2}b\left(\mu_1^2 - \mu_2^2\right) \\ - \frac{1}{3}b\theta\left(\mu_1^3 - \mu_2^3\right) + a\theta\mu_2\left(\mu_1 - \mu_2\right) - \rho p\theta\mu_2\left(\mu_1 - \mu_2\right) + \frac{1}{2}b\theta\mu_2\left(\mu_1^2 - \mu_2^2\right) \end{array}\right] + \left(a\mu_1 - \rho p\mu_1 + \frac{1}{2}b\mu_1^2\right) + \left[a\left(T - t_0\right) - \rho p\left(T - t_0\right) + \frac{1}{2}b\left(T^2 - t_0^2\right)\right]. \tag{20}$$

Putting value of Q_1 and Q in equation (11), we have

$$I(t) = Q_1 - dQ - (at - \rho pt + \frac{1}{2}bt^2).$$

$$I(t) = \frac{(1-d)}{\left[1+\theta(\mu_1-\mu_2)\right]}\begin{bmatrix} a(t_0-\mu_2) - \rho p(t_0-\mu_2)+\frac{1}{2}b(t_0^2-\mu_2^2)+\frac{1}{6}a\theta(t_0^3-\mu_2^3)-\frac{1}{6}\rho p\theta(t_0^3-\mu_2^3) \\ +\frac{1}{8}b\theta(t_0^4-\mu_2^4) - \frac{1}{2}a\theta\mu_2^2(t_0-\mu_2)+\frac{1}{2}\rho p\theta\mu_2^2(t_0-\mu_2)-\frac{1}{4}b\theta\mu_2^2(t_0^2-\mu_2^2) \\ -a(\mu_1-\mu_2)+\rho p(\mu_1-\mu_2)-\frac{1}{2}a\theta(\mu_1^2-\mu_2^2)+\frac{1}{2}\rho p\theta(\mu_1^2-\mu_2^2)-\frac{1}{2}b(\mu_1^2-\mu_2^2) \\ -\frac{1}{3}b\theta(\mu_1^3-\mu_2^3)+a\theta\mu_2(\mu_1-\mu_2)-\rho p\theta\mu_2(\mu_1-\mu_2)+\frac{1}{2}b\theta\mu_2(\mu_1^2-\mu_2^2) \end{bmatrix}$$
$$+(1-d)\left(a\mu_1-\rho p\mu_1+\frac{1}{2}b\mu_1^2\right)-d\left[a(T-t_0)-\rho p(T-t_0)+\frac{1}{2}b(T^2-t_0^2)\right]$$
$$-\left(at-\rho pt+\frac{1}{2}bt^2\right). \qquad (21)$$

Based on the assumptions and descriptions of the model, the total annual relevant profit (π), include the following elements:

(i) Ordering cost (OC) = A (22)

(ii) Screening cost (SrC) = zQ (23)

(iii) Holding Cost (HC) is:

$$HC = \int_0^{t_0}(x+yt)I(t)e^{-Rt}dt$$
$$= \int_0^{t_1}(x+yt)I(t)e^{-Rt}dt + \int_{t_1}^{\mu_1}(x+yt)I(t)e^{-Rt}dt + \int_{\mu_1}^{\mu_2}(x+yt)I(t)e^{-Rt}dt + \int_{\mu_2}^{t_0}(x+yt)I(t)e^{-Rt}dt \qquad (24)$$

(iv) Deterioration Cost (DC) is:

$$DC = c\left(\int_{\mu_1}^{\mu_2}\theta I(t)e^{-Rt}dt + \int_{\mu_2}^{t_0}\theta tI(t)e^{-Rt}dt\right) \qquad (25)$$

(v) Shortages Cost (SC) is:

$$SC = -c_2\int_{t_0}^{T} I(t)e^{-Rt}dt \qquad (26)$$

(vi) Sales Revenue (SR) is:

$$SR = \left(p\int_0^T(a+bt-\rho p)e^{-Rt}dt + p_d dQ\right). \qquad (27)$$

To determine the interest earned, there will be two cases i.e.

Case I: ($0\le M\le t_0$) and Case II: ($M> t_0$).

Case I: ($0\le M\le t_0$): In this case the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_0.

(vi) Interest earned per cycle:

$$IE_1 = pI_e \int_0^M (a + bt - \rho p) t\, e^{-Rt} dt \qquad (28)$$

Case II: (M> t_0):

In this case, the retailer earns interest on the sales revenue up to the permissible delay period. So

(vii) Interest earned up to the permissible delay period is:

$$IE_2 = p\, I_e \left[\int_0^{t_0} (a + bt - \rho p) t\, e^{-Rt} dt + (a + bt_0 - \rho p) t_0 (M - t_0) \right] \qquad (29)$$

To determine the interest payable, there will be four cases i.e.

Interest payable per cycle for the inventory not sold after the due period M is

Case I: (0≤M≤μ_1):

$$\text{(viii) } IP_1 = cI_p \int_M^{t_0} I(t) e^{-Rt} dt = cI_p \left(\int_M^{\mu_1} I(t) e^{-Rt} dt + \int_{\mu_1}^{\mu_2} I(t) e^{-Rt} dt + \int_{\mu_2}^{t_0} I(t) e^{-Rt} dt \right) \qquad (30)$$

Case II: (μ_1≤M≤ μ_2):

$$\text{(ix) } IP_2 = cI_p \int_M^{t_0} I(t) e^{-Rt} dt = cI_p \left(\int_M^{\mu_2} I(t) e^{-Rt} dt + \int_{\mu_2}^{t_0} I(t) e^{-Rt} dt \right) \qquad (31)$$

Case III: (μ_2≤M≤t_0):

$$\text{(x) } IP_3 = cI_p \int_M^{t_0} I(t) e^{-Rt} dt \qquad (32)$$

Case IV: (M>t_0):

$$\text{(xi) } IP_4 = 0 \qquad (33)$$

(by neglecting higher powers of θ and R)

The total profit (π_i), i=1,2,3 and 4 during a cycle consisted of the following:

$$\pi_i = \frac{1}{T}[SR - OC - SrC - HC - DC - SC - IP_i + IE_i] \qquad (34)$$

Substituting values from equations (22) to (33) in equation (34), we get total profit per unit. Putting $\mu_1 = v_1 t_0$, $\mu_2 = v_2 t_0$ in equation (34), and value of t_1 and Q in equation (34), we get profit in terms of t_0, T and p for the four cases as under:

$$\pi_1 = \frac{1}{T}[SR - OC - SrC - HC - DC - SC - IP_1 + IE_1] \qquad (35)$$

$$\pi_2 = \frac{1}{T}[SR - OC - SrC - HC - DC - SC - IP_2 + IE_1] \qquad (36)$$

$$\pi_3 = \frac{1}{T}[SR - OC - SrC - HC - DC - SC - IP_3 + IE_1] \qquad (37)$$

$$\pi_4 = \frac{1}{T}[SR - OC - SrC - HC - DC - SC - IP_4 + IE_2] \qquad (38)$$

Differentiating equations (35) to (38) with respect to t_0, T and p and equate it to zero, we have

$$\text{i.e. } \frac{\partial \pi_i(t_0,T,p)}{\partial t_0} = 0,\ \frac{\partial \pi_i(t_0,T,p)}{\partial T} = 0,\ \frac{\partial \pi_i(t_0,T,p)}{\partial p} = 0,\ i = 1,2,3,4 \qquad (39)$$

provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial \pi_i^2(t_0,T,p)}{\partial t_0^2} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial t_0 \partial T} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial t_0 \partial p} \\ \frac{\partial \pi_i^2(t_0,T,p)}{\partial T \partial t_0} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial T^2} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial T \partial p} \\ \frac{\partial \pi_i^2(t_0,T,p)}{\partial p \partial t_0} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial p \partial T} & \frac{\partial \pi_i^2(t_0,T,p)}{\partial p^2} \end{vmatrix} > 0, \quad i = 1,2,3,4. \qquad (40)$$

4. NUMERICAL EXAMPLE

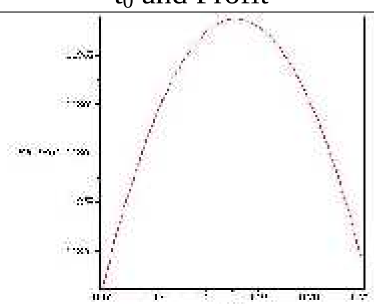
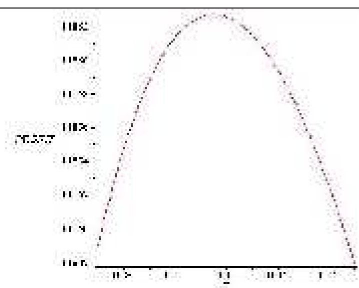
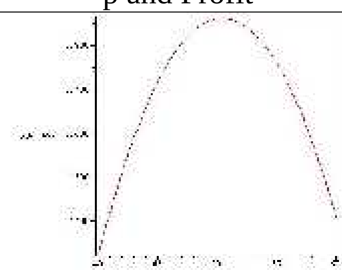
Case I: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, c_2 = Rs. 12, R = 0.06, Ie = 0.12, Ip=0.15, M=0.04 in appropriate units. The optimal value of t_0* = 0.1857, T* =0.3150, p*=50.4183, Profit*= Rs. 11862.7475 and optimum order quantity Q* = 78.1517.

Case II: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, c_2 = Rs. 12, R = 0.06, Ie = 0.12, Ip=0.15, M=0.07 in appropriate units. The optimal value of t_0* = 0.1889, T* =0.3134, p*=50.3907, Profit*= Rs. 11882.9813 and optimum order quantity Q* = 77.8004.

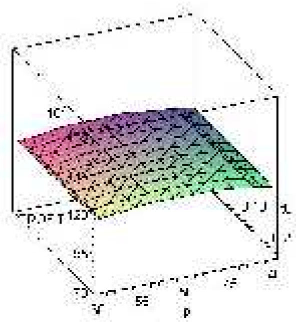
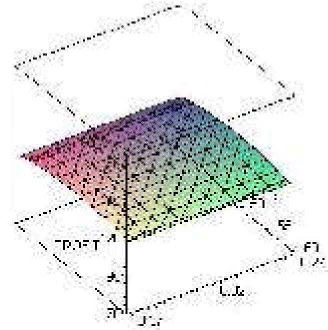
Case III: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, c_2 = Rs. 12, R = 0.06, Ie = 0.12, Ip=0.15, M=0.14 in appropriate units. The optimal value of t_0* = 0.1970, T* =0.3038, p*=50.3422, Profit*= Rs. 11936.2723 and optimum order quantity Q* = 75.4983.

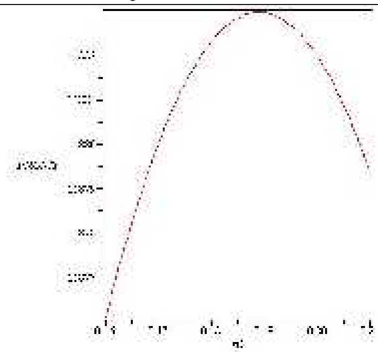
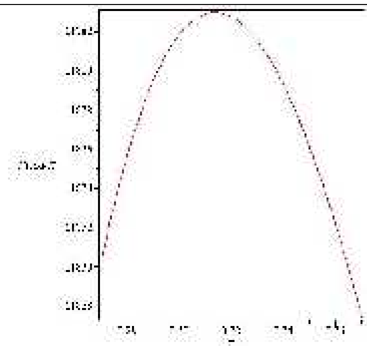
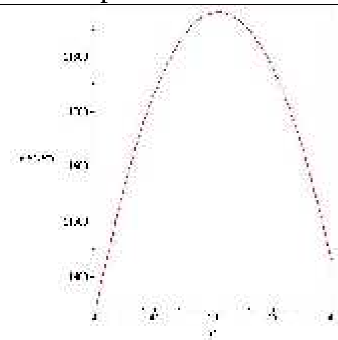
Case IV: Considering A= Rs.100, a = 500, b=0.05, c=Rs. 25, p_d = 15, d= 0.02, z = 0.40, λ=10000, θ=0.05, x = Rs. 5, y=0.05, v_1=0.30, v_2 = 0.50, c_2 = Rs. 12, R = 0.06, Ie = 0.12, Ip=0.15, M=0.25 in appropriate units. The optimal value of t_0* = 0.2087, T* =0.2832, p*=50.3109, Profit*= Rs. 12045.8900 and optimum order quantity Q* = 70.4362.

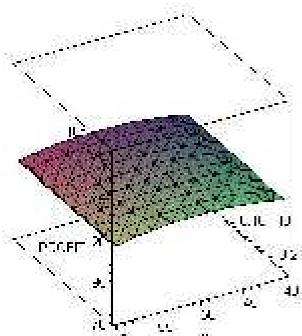
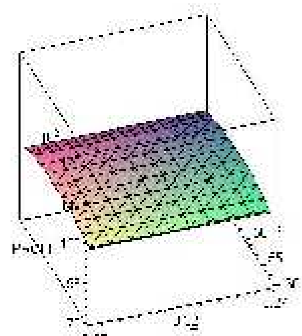
The second order conditions given in equation (40) are also satisfied. The graphical representation of the concavity of the profit function is also given.

Case I		
t_0 and Profit	T and Profit	p and Profit
 Graph 1	 Graph 2	 Graph 3

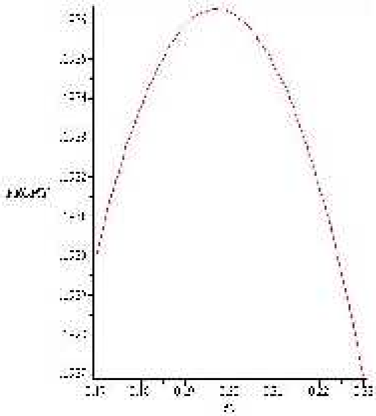
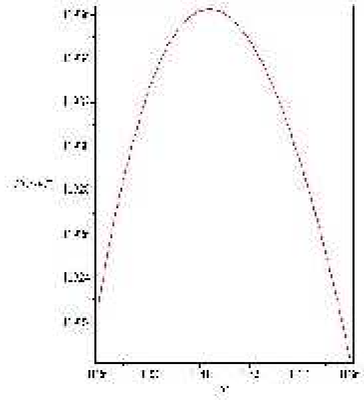
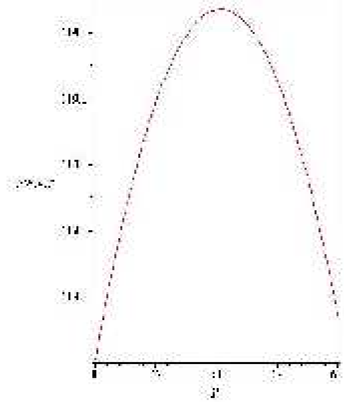
Case I	
t_0, p and Profit	T, p and Profit

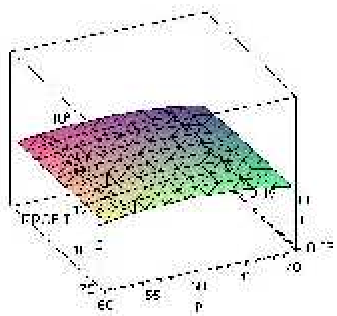
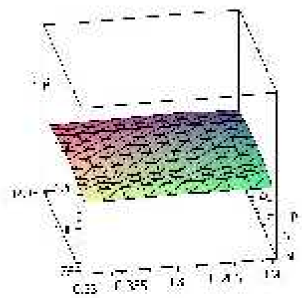
 Graph 4	 Graph 5

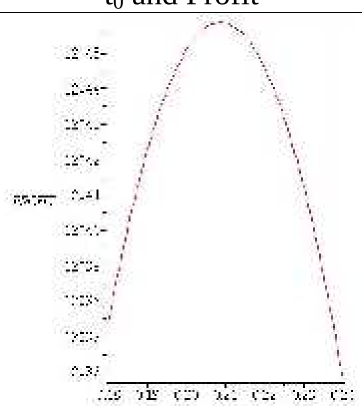
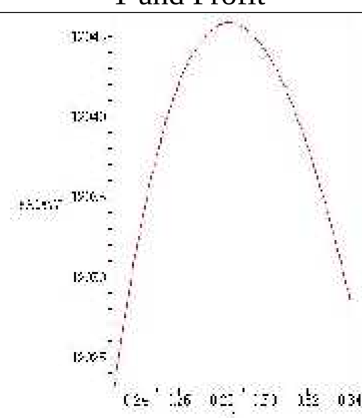
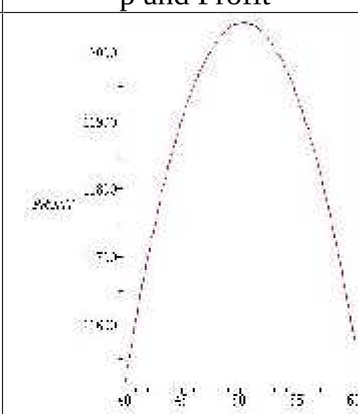
Case II		
t_0 and Profit	T and Profit	p and Profit
 Graph 6	 Graph 7	 Graph 8

Case II	
t_0, p and Profit	T, p and Profit
 Graph 9	 Graph 10

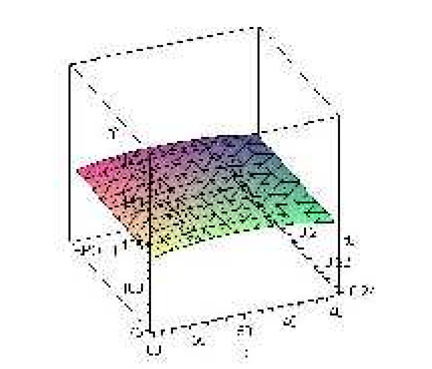
Case III		
t_0 and Profit	T and Profit	p and Profit

 Graph 11	 Graph 12	 Graph 13

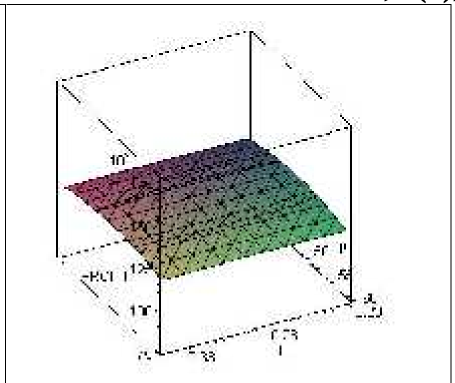
Case III	
t_0, p and Profit	T, p and Profit
 Graph 14	 Graph 15

Case IV		
t_0 and Profit	T and Profit	p and Profit
 Graph 16	 Graph 17	 Graph 18

Case IV	
t_0, p and Profit	T, p and Profit



Graph 19



Graph 20

5. SENSITIVITY ANALYSIS

On the basis of the data given in example above we have studied the sensitivity analysis by changing the following parameters one at a time and keeping the rest fixed.

Table 1
Case I
Sensitivity Analysis

Para-meter	%	t_0	T	p	Profit	Q
a	+20%	0.1640	0.2765	60.3691	17277.2080	82.4953
	+10%	0.1741	0.2944	55.3920	14444.6223	80.4409
	-10%	0.1992	0.3390	45.4491	9531.6902	75.7776
	-20%	0.2151	0.3675	40.4856	7451.5989	72.6742
x	+20%	0.1743	0.3094	50.4368	11850.0799	76.7274
	+10%	0.1798	0.3121	50.4278	11856.2405	77.4140
	-10%	0.1920	0.3181	50.4082	11869.6319	78.9157
	-20%	0.1987	0.3215	50.3975	11976.9285	79.8056
θ	+20%	0.1848	0.3145	50.4196	11861.8114	78.0365
	+10%	0.1852	0.3148	50.4189	11862.2785	78.1066
	-10%	0.1861	0.3152	50.4177	11863.2187	78.1966
	-20%	0.1866	0.3155	50.4170	11863.6919	78.2665
A	+20%	0.2028	0.3455	50.4574	11802.1862	85.6622
	+10%	0.1944	0.3306	50.4383	11831.7693	81.9921
	-10%	0.1765	0.2986	50.3973	11895.3376	74.1112
	-20%	0.1667	0.2813	50.3752	11929.8188	69.8455
ρ	+20%	0.1918	0.3259	42.0988	9798.9264	80.6946
	+10%	0.1889	0.3208	45.8802	10736.9307	79.5116
	-10%	0.1819	0.3083	55.9654	13239.0358	76.5643
	-20%	0.1776	0.3006	62.9000	14959.8085	74.7242
λ	+20%	0.1857	0.3150	50.4182	11862.7684	78.1518
	+10%	0.1857	0.3150	50.4182	11862.7589	78.1518
	-10%	0.1857	0.3150	50.4184	11862.7336	78.1516
	-20%	0.1857	0.3150	50.4184	11862.7162	78.1516
	+20%	0.1794	0.3041	50.4044	11839.8698	75.4664
	+10%	0.1825	0.3094	50.4112	11851.2094	76.7722

R	-10%	0.1891	0.3209	50.4258	11874.4950	79.6046
	-20%	0.1927	0.3272	50.4338	11886.4635	81.1557
M	+20%	0.1869	0.3146	50.4110	11867.6833	78.0648
	+10%	0.1863	0.3148	50.4146	11865.1968	78.1083
	-10%	0.1851	0.3152	50.4220	11860.3356	78.1950
	-20%	0.1844	0.3154	50.4258	11857.9610	78.2381
c_2	+20%	0.1942	0.3075	50.4363	11848.9731	76.2703
	+10%	0.1902	0.3110	50.4279	11855.4466	77.1480
	-10%	0.1805	0.3197	50.4073	11871.0475	79.3311
	-20%	0.1745	0.3252	50.3945	11880.5699	80.7121

Table 2
Case II
Sensitivity Analysis

Para-meter	%	t_0	T	p	Profit	Q
a	+20%	0.1668	0.2738	60.3416	17304.2360	54.3312
	+10%	0.1771	0.2923	55.3644	14468.0760	65.2843
	-10%	0.2026	0.3379	45.4214	9549.0256	92.2902
	-20%	0.2188	0.3668	40.4578	7466.3247	109.3025
x	+20%	0.1773	0.3079	50.4105	11869.8051	76.3980
	+10%	0.1829	0.3105	50.4009	11876.2117	77.0612
	-10%	0.1953	0.3165	50.3799	11890.1463	78.5909
	-20%	0.2022	0.3198	50.3684	11897.7429	79.4330
θ	+20%	0.1879	0.3130	50.3920	11882.1556	77.7104
	+10%	0.1884	0.3132	50.3913	11882.5669	77.7552
	-10%	0.1894	0.3136	50.3901	11883.3988	77.8452
	-20%	0.1900	0.3137	50.3894	11883.8192	77.8654
A	+20%	0.2061	0.3441	50.4292	11822.1424	85.3619
	+10%	0.1977	0.3290	50.4103	11851.8524	81.6440
	-10%	0.1797	0.2969	50.3701	11915.7524	73.7319
	-20%	0.1699	0.2794	50.3484	11950.4541	69.4134
ρ	+20%	0.1954	0.3249	42.0716	9817.7656	80.5028
	+10%	0.1923	0.3195	45.8528	10756.3939	79.2401
	-10%	0.1850	0.3063	55.9375	13260.2438	76.1085
	-20%	0.1804	0.2981	62.8718	14982.2839	74.1386
λ	+20%	0.1889	0.3134	50.3907	11882.9942	77.8004
	+10%	0.1889	0.3134	50.3907	11882.9887	77.8004
	-10%	0.1889	0.3134	50.3907	11882.9727	77.8004
	-20%	0.1889	0.3134	50.3907	11882.9620	77.8004
R	+20%	0.1827	0.3025	50.3772	11860.2003	75.1129
	+10%	0.1857	0.3078	50.3837	11871.4917	76.4199
	-10%	0.1923	0.3193	50.3980	11894.6800	79.2546
	-20%	0.1958	0.3255	50.4058	11906.5993	80.7821
	+20%	0.1907	0.3121	50.3794	11892.8174	77.4968
	+10%	0.1898	0.3128	50.3850	11887.8418	77.6611

M	-10%	0.1880	0.3139	50.3966	11878.2355	77.9146
	-20%	0.1870	0.3145	50.4027	11873.6037	78.0531
c_2	+20%	0.1971	0.3058	50.4069	11870.1831	75.8960
	+10%	0.1933	0.3093	50.3993	11876.1898	76.7730
	-10%	0.1839	0.3181	50.3807	11890.7244	78.9789
	-20%	0.1781	0.3237	50.3691	11899.6374	80.3834

Table 3
Case III
Sensitivity Analysis

Para-meter	%	t_0	T	p	Profit	Q
a	+20%	0.1723	0.2600	60.2939	17381.5725	77.6792
	+10%	0.1840	0.2806	55.3162	14532.5220	76.7861
	-10%	0.2118	0.3301	45.3724	9592.6995	73.7296
	-20%	0.2290	0.3608	40.4082	7501.7368	71.4992
x	+20%	0.1853	0.2985	50.3661	11921.4236	74.1386
	+10%	0.1909	0.3010	50.3545	11928.6508	74.7803
	-10%	0.2035	0.3067	50.3291	11944.3224	76.2432
	-20%	0.2104	0.3099	50.3153	11952.8395	77.0646
θ	+20%	0.1960	0.3033	50.3439	11935.1757	75.3840
	+10%	0.1965	0.3035	50.3430	11935.7228	75.4288
	-10%	0.1975	0.3040	50.3413	11936.8241	75.5431
	-20%	0.1979	0.3042	50.3405	11937.3783	75.5876
A	+20%	0.2146	0.3351	50.3781	11873.6644	83.2232
	+10%	0.2060	0.3198	50.3605	11904.2015	79.4485
	-10%	0.1875	0.2869	50.3231	11970.1324	71.3229
	-20%	0.1773	0.2689	50.3033	12006.1171	66.8717
ρ	+20%	0.2050	0.3180	42.0249	9864.5795	78.8905
	+10%	0.2012	0.3113	45.8053	10806.0972	77.2957
	-10%	0.1920	0.2950	55.8879	13318.0990	56.8724
	-20%	0.1862	0.2846	62.8208	15046.1227	52.9518
λ	+20%	0.1970	0.3038	50.3421	11936.2914	75.4985
	+10%	0.1970	0.3038	50.3421	11936.2828	75.4985
	-10%	0.1970	0.3038	50.3422	11936.2596	75.4983
	-20%	0.1970	0.3038	50.3423	11936.2437	75.4982
R	+20%	0.1910	0.2933	50.3304	11914.0803	72.9043
	+10%	0.1939	0.2984	50.3361	11925.0807	74.1644
	-10%	0.2002	0.3095	50.3486	11947.6655	76.9215
	-20%	0.2037	0.3155	50.3554	11959.2714	78.3875
M	+20%	0.1990	0.2985	50.3282	11961.3255	74.2047
	+10%	0.1981	0.3013	50.3347	11948.5447	74.8896
	-10%	0.1958	0.3060	50.3505	11924.4968	76.0311
	-20%	0.1944	0.3080	50.3596	11913.2083	76.5126
	+20%	0.2036	0.2968	50.3536	11926.5525	73.7481
	+10%	0.2005	0.3000	50.3483	11931.1112	74.5481

c_2	-10%	0.1930	0.3081	50.3351	11942.1653	76.5742
	-20%	0.1883	0.3133	50.3268	11948.9597	77.8754

Table 4
Case IV
Sensitivity Analysis

Para-meter	%	t_0	T	p	Profit	Q
a	+20%	0.1873	0.2348	60.2749	17550.5048	70.1898
	+10%	0.1971	0.2577	55.2907	14669.5674	70.5672
	-10%	0.2226	0.3122	45.3359	9678.9664	69.7999
	-20%	0.2399	0.3459	40.3667	7568.4232	68.6288
x	+20%	0.1988	0.2808	50.3407	12027.8079	69.6418
	+10%	0.2036	0.2817	50.3262	12036.6509	70.0382
	-10%	0.2140	0.2848	50.2945	12055.5532	70.8612
	-20%	0.2196	0.2865	50.2770	12065.6707	71.3133
θ	+20%	0.2078	0.2829	50.3130	12044.5578	70.3729
	+10%	0.2083	0.2830	50.3120	12045.2228	70.3921
	-10%	0.2091	0.2833	50.3098	12046.5596	70.4555
	-20%	0.2095	0.2835	50.3087	12047.2315	70.4995
A	+20%	0.2241	0.3139	50.3376	11978.8971	78.0342
	+10%	0.2166	0.2989	50.3244	12011.5329	74.3230
	-10%	0.2003	0.2665	50.2970	12082.2721	66.2993
	-20%	0.1913	0.2488	50.2828	12121.0835	61.9117
ρ	+20%	0.2170	0.3023	41.9936	9954.9352	75.0651
	+10%	0.2130	0.2933	45.7738	10905.0356	72.8899
	-10%	0.2040	0.2715	55.8573	13441.3231	67.8573
	-20%	0.1990	0.2578	62.7921	15187.2108	64.2178
λ	+20%	0.2087	0.2832	50.3108	12045.9064	70.4364
	+10%	0.2087	0.2832	50.3109	12045.8989	70.4362
	-10%	0.2087	0.2832	50.3108	12045.8792	70.4362
	-20%	0.2087	0.2832	50.3110	12045.8656	70.4361
R	+20%	0.2039	0.2740	50.3033	12024.9816	68.1572
	+10%	0.2062	0.2785	50.3070	12035.3507	69.2720
	-10%	0.2112	0.2881	50.3150	12056.6082	71.6497
	-20%	0.2140	0.2933	50.3195	12067.5145	72.9372
M	+20%	0.2170	0.2743	50.3106	12103.1664	68.2318
	+10%	0.2130	0.2789	50.3100	12074.0181	69.3724
	-10%	0.2041	0.2870	50.3130	12018.7375	71.3739
	-20%	0.1994	0.2903	50.3161	11992.5227	72.1859
c_2	+20%	0.2120	0.2769	50.3181	12040.8248	68.8635
	+10%	0.2105	0.2798	50.3148	12043.1962	69.5874
	-10%	0.2066	0.2871	50.3063	12048.9777	71.4103
	-20%	0.2041	0.2917	50.3008	12052.5533	72.5594

From the table we observe that as parameter a increases/ decreases average total profit and increases/ decreases for all four case, and optimum order quantity also increases/ decreases for case I, III and decreases/ increases for cases II and IV respectively.

Also, we observe that with increase and decrease in the value of x, R and c_2, there is corresponding decrease/ increase in total profit and optimum order quantity for all four cases.

From the table we observe that as parameter A and ρ increases/ decreases average total profit decreases/ increases and optimum order quantity increases/ decreases for all four cases.

From the table we observe that as parameter M increases/ decreases average total profit increases/ decreases and there is very minor change in optimum order quantity for all four cases.

From the table we observe that as parameter λ increases/ decreases, there is very minor increase/decrease in average total profit and almost no change in optimum order quantity for all four cases.

From the table we observe that as parameter θ increases/ decreases, there is corresponding decrease/ increase in total profit and very minor decrease/ increase in optimum order quantity for all four cases.

6. CONCLUSION

In this paper, we have developed an inventory model for deteriorating items with price and time dependent demand with different deterioration rates and shortages under inflation and permissible delay in payments. Sensitivity with respect to parameters have been carried out. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of profit.

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